Periodic Boundary Conditions in Natural Element Method

M. M. Afonso[1], B. M. F. Gonçalves[1], E. H. R. Coppoli[1], M. A. O. Scroeder[1], R. S. Alípio[1], B. Ramdane[2] and Y. Marechal[2]

[1]Electrical Engineering Postgraduate Program - CEFET-MG – UFSJ, Minas Gerais, Brazil, marciomatias@des.cefetmg.br
[2]Electrical Engineering Laboratory of Grenoble – G2ELAB, Grenoble, France, brahim.ramdane@g2elab.grenoble-inp.fr

In this paper, periodic and anti-periodic boundary conditions are introduced in the Natural Element Method (NEM). These boundaries conditions are important because they allow to explore the inherent symmetry of the electromagnetic devices. It is shown that with these techniques NEM can now easily take advantage of electrical machines symmetry. The proposed approach is evaluated and compared with the traditional Finite Element Method (FEM).

Index Terms **— Periodic Boundary Conditions, Natural Element Method, Electrical Machines.**

I. Introduction

THE meshless methods are increasingly being used in electromagnetic field computations [1]. In these methods a cloud of nodes without connectivity relations that covers the domain is used to solve the problem [2]. Due to its characteristics, these methods are well suited to solve problems involving moving parts like electrical machines [1]-[2]. However, in this kind of problem it is important to consider the symmetry in order to reduce the number of unknown. Periodic and anti-periodic boundary conditions are useful techniques to explore the inherent symmetry of electrical machines [1].

It is well known that meshless methods provides high accuracy solutions but present some difficulties to handle boundary and interface conditions [3]. To eliminate these drawbacks, the natural element method (NEM) was proposed [2]. The NEM approach is based on the Voronoï diagram and the natural neighbors concept. The main interest of NEM lies in its interpolation property which allows enforcing essential boundary conditions in an easy way as with the FEM [2]. Also, this method retains the natural capacity of treating heterogeneous domain and present similar numerical behavior with a better convergence compared to FEM in some cases [3].

As the NEM shape functions verify the Kronecker delta property, the imposition of periodic and anti-periodic boundary conditions can be done with the same techniques used by FEM [4]. Therefore, the aim of this paper is to introduce the periodic and anti-periodic boundary conditions in the natural element method. The proposed approach is applied to periodic electromagnetic device and the result is compared with traditional FEM.

II. Problem Formulation

For the purpose of analysis, consider the periodic structure shown by Fig. 1. It is characterized by a geometric replication of the picked out Ω domain. If there are coils and/or permanent magnets oriented in the same direction, the potentials on line *c* are identical to those on line *d* [4].

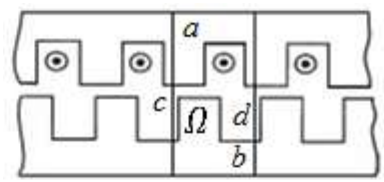


Fig. 1: Periodic Structure: Only the repetitive Ω domain delimited by lines *a*, *b*, *c* and *d*. Dot in the circles indicate current entrance [4].

An anti-periodic structure is similar to the aforementioned case except that the source (current or permanent magnet) has alternately opposing directions [4]. If a 2D magnetostatic phenomenon occurs in Ω due to current circulation so, its Galerkin *weak formulation* can be written as [4]:

$$\int_{\Omega}\left[\nabla.\left(\frac{1}{\mu}\nabla A\right)+J\right]w\,d\Omega=0 \qquad (1)$$

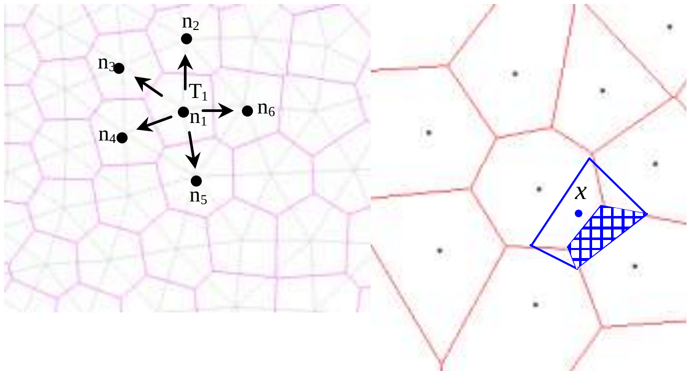
In (1) Ω is the problem domain surrounded by the surface Γ (given by the lines *a*, *b*, *c* and *d*), A is an approximation for the scalar component of the magnetic potential vector, μ is the permeability, J is the current density whose distribution is assumed to be known in Ω and w is the scalar weight function. Many numerical techniques can be used in the evaluation of (1). However, this paper is focused on NEM.

III. The Natural Element Method (NEM)

The natural element method uses the concept of natural neighbors which is based on the construction of Voronoï diagram on a cloud of nodes. This diagram subdivides the studied domain into a set of polygons which defines the natural neighbors of the node in its center. The Delaunay triangulation, which is the dual of the Voronoï diagram, is constructed by connecting the nodes whose Voronoï cells have common boundaries (Fig. 2 (a))[5].

Based on the Voronoï diagram, the NEM shape function can be calculated. In the literature, several formulas are used to calculate this shape function [2]. Among the most used, are the

Sibson functions which may be determined in analogy with classical FEM shape functions as the ratio of surfaces in the case of triangles [5]. Thus, at a point x shown by Fig. 2 (b), the Sibson shape function is given by (2) where $S(x)$ is the area of Voronoï cell and $S_i(x)$ represents the subarea of Voronoï cell linked to the natural neighbor n_i (blue surface) [5].



(a) (b)

Fig. 2. (a) Representation of Voronoï diagram (pink and black colors) and associated Delaunay triangulation (gray color). One node, its Voronoï cell and its five natural neighbors are highlighted. (b) NEM shape function computation.

$$\phi_i(x) = \frac{S_i(x)}{S(x)} \tag{2}$$

As (2) verifies the same properties of FEM shape functions as Kronecker delta, interpolation and partition of unity are verified [2, 5]. Thus, the potential can be written as follows:

$$A(x) = \sum_{i=1}^{N} A_i \ \phi_i(x) \tag{3}$$

where N is the number of natural neighbors visible from point x. Due to the inherent meshless character, (3) can be computed in arbitrary clouds of nodes and these nodes can move freely on the background without any geometrical restriction [2]. To treat the materials discontinuity a constrained Delaunay triangulation associated with a visibility criterion is used. This procedure constitutes an extension of NEM to the constrained Natural Element Method (C-NEM) [3].

IV. Results

Consider the structure showed at Fig. 3 to validate the proposed approach. The circles are aluminium conductors carrying a current density in the same direction (exiting the sheet plane) of $1 MA/m^2$ and involved by an iron material with $\mu_r = 7000$.

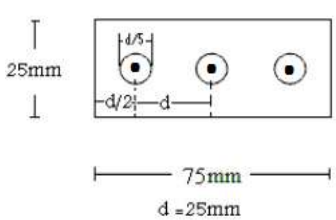


Fig.3. Periodic structure [2].

The periodic boundary conditions are set at the left and right boundaries while zero Dirichlet ones are set at the top and bottom boundaries. Fig. 4 shows the resulting induction flux distribution for a 1120 nodes discretization.

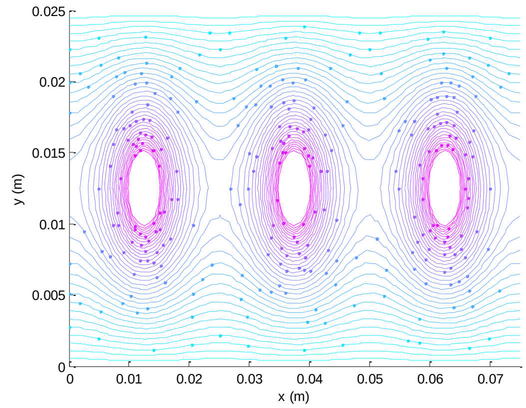


Fig.4. The resulting induction flux distribution [2].

The result is also validated by comparison with the FEM ones obtained with the same number of nodes. Fig. 5 shows the potential distribution on the boundary of the periodic structure. The analysis of these solutions has demonstrated the applicability and accuracy of NEM to treat periodicity. In the full paper, more details of the NEM and the periodicity treatment will be discussed. Applications of this technique to solve anti-periodicity and electric machine will be performed.

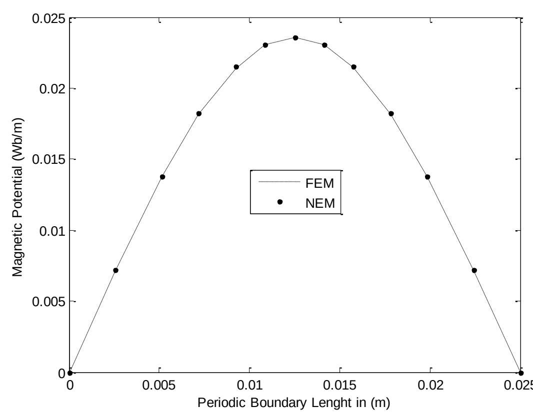


Fig. 5. Potential distribution along the periodic boundary.

V. Acknowledgement

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VI. References

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